



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\therefore 1521 = 13^2 + 34^2 + 14^2 = 13^2 + 26^2 + 26^2 = 34^2 + 2^2 + 19^2 = 26^2 + 22^2 + 19^2 \\ = 26^2 + 2^2 + 29^2 = 14^2 + 22^2 + 29^2 = 14^2 + 35^2 + 10^2.$$

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let  $1521 = m^2$ , and  $x^2, y^2$ , and  $z^2$  represent the three squares; then  $x^2 + y^2 + z^2 = m^2$ , and  $x^2 = m^2 - (y^2 + z^2) = m^2 - 2pm + p^2$ .  $2pm = p^2 + y^2 + z^2$ . Let  $y = tp$  and  $z = sp$ , then  $2m = p(s^2 + t^2 + 1)$ . Restoring the value of  $m$ ,  $p^2 = 78/(s^2 + t^2 + 1)$ , in which  $s$  and  $t$  may be any rational numbers. Take  $s = 1, t = 1$ , then  $p = 26$ ;  $x = m - t = 13$ ,  $y = tp = 26$  and  $z = sp = 26$ , and  $13^2 + 26^2 + 26^2 = 1521$ . Take  $s = 2$ , and  $t = 1$ ,  $p = 13$ ;  $x = 26$ ,  $y = 13$ ,  $z = 26$ . Take  $s = 3$ ,  $t = 4$ , then  $p = 3$ ;  $x = 36$ ,  $y = 12$ ,  $z = 9$ . Take  $s = 2$ ,  $t = 3$ ,  $p = \frac{39}{7}$ ;  $x = \frac{234}{7}$ ,  $y = \frac{78}{7}$ ,  $z = \frac{117}{7}$ , and  $[(234)^2 + (78)^2 + (117)^2]/49 = 1521$ .

While this is a solution of the question read *literally*, of course, I understand that the proposer intends to call for integral numbers; but I have obtained seven integral results, only *by trial*.

Also solved by SYLVESTER ROBINS, and G. B. M. ZERR.

66. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find two cubic proper fractions whose product is a square proper fraction. Can a general solution be made?

Solution by E. L. SHERWOOD, A. M., Superintendent City Schools, West Point, Miss.; CHARLES CARROLL CROSS, Libertytown, Md.; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Let  $a^6/b^6, c^6/d^6$  be the fractions,  $a < b, c < d$ .

Then  $a^6c^6/b^6d^6 = (a^3c^3/b^3d^3)^2$ .

Let  $a = 1, b = 2, c = 2, d = 3$ .

$\therefore a^6/b^6 = \frac{1}{64}, c^6/d^6 = \frac{64}{27}$ ;

$\therefore a^6/b^6 = (\frac{1}{4})^3, c^6/d^6 = (\frac{4}{9})^3; a^6c^6/b^6d^6 = (\frac{1}{27})^2$ .

Other fractions can easily be found.

Also solved by J. H. DRUMMOND.

67. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find (1) four consecutive numbers whose sum is a square, and (2) four consecutive numbers the sum of whose squares is a square.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let  $x - 1, x, x + 1$  and  $x + 2$  = four consecutive integers.

(1) Then their sum  $= 4x + 2 = 2(2x + 1) = 2$  times an odd number. But this result can never be a square.  $\therefore$  The sum of four consecutive integers can not be a square.

(2) The sum of their squares  $= 4x^2 + 4x + 6 = 2[2(x^2 + x + 1) + 1] = 2$  times an odd number, which result can never be a square.  $\therefore$  The sum of the squares of four consecutive integers can not be a square.

If, however, four consecutive numbers may be considered as four fractions whose denominators are the same number and equal to 2 times a square, and

whose numerators are consecutive integers, then we are able to fulfill the *first part* of the problem.

Of any four consecutive integers we have shown that their sum is *2 times an odd number*. Now when this odd number is a *square*, we can find four consecutive fractions whose sum is a square, by making the denominators =  $2m^2$  and the respective numerators =  $2n(n+1)-1$ ,  $2n(n+1)$ ,  $2n(n+1)+1$ , and  $2n(n+1)+2$ . Whence we have  $\{[2n(n+1)-1]/2m^2\} + \{[2n(n+1)]/2m^2\} + \{[2n(n+1)+1]/2m^2\} + \{[2n(n+1)+2]/2m^2\} = (2n+1)^2/m^2$ .

When  $n=m=1$ , we find  $\frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \frac{6}{2} = 3^2$ . When  $n=1$  and  $m=2$ , we have  $\frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} = (\frac{3}{2})^2$ . When  $n=2$  and  $m=1$ , we find  $\frac{11}{2} + \frac{12}{2} + \frac{13}{2} + \frac{14}{2} = 5^2$ , etc.

### II. Solution by CHARLES CARROLL CROSS, Libertytown, Md.

(1) Combining the consecutive numbers we find that  $1+2+3+4$ ,  $5+6+7+8$ ,  $6+7+8+9$ ,  $7+8+9+0$ , and  $0+1+2+3$  are the only combinations whose terminal figure produces the terminal figure of a square. The first and third combinations can never produce a square number, because a square number whose terminal figure is 0 is always preceded by 0. The second and last combinations cannot produce a square number, because a square number whose terminal figure is 6 is always preceded by an odd number. The fourth combination can never be a square number, because a square number whose terminal figure is 4 is always preceded by an even number. Hence (1) is incorrect.

(2) In the *Mathematical Visitor*, Vol. I, No. 5, page 156, Dr. Martin has shown that the sum of three, of four, and of five consecutive squares, cannot be a square number. Hence (2) is also incorrect. [See also *Mathematical Magazine*, Vol. II, No. 6, page 92.]

### III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

(1) Let  $x$ ,  $x+1$ ,  $x+2$ ,  $x+3$  be the numbers.

$$\therefore 4x+6=a^2, \text{ or } x=(a^2-6)/4.$$

$\therefore (a^2-6)/4, (a^2-2)/4, (a^2+2)/4, (a^2+6)/4$  are the numbers.

$$(2) 4x^2+12x=b^2-14.$$

$\therefore x=\frac{1}{2}\sqrt{(a^2-5)}-\frac{3}{2}$ , where  $(a^2-5)$  must be a square.

Let  $a=3$ , then  $-\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$ , are the numbers.

Let  $a=2\frac{1}{2}$ , then  $-\frac{7}{6}$ ,  $-\frac{1}{6}$ ,  $\frac{5}{6}$ ,  $\frac{11}{6}$  are the numbers.

And so for other values of  $a$ .

Also solved by EDWARD R. ROBBINS, and J. H. DRUMMOND.